basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

EXEMPLAR 2013

MEMORANDUM

MARKS: 150

This memorandum consists of 13 pages.

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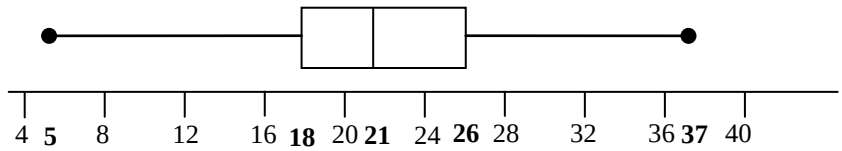
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

QUESTION 1

1.1	Mean = $\frac{\sum_{i=1}^{n} x_i}{n} = \frac{408}{19} = 21{,}47$	✓ $\frac{408}{19}$ ✓answer (2)
1.2	Standard deviation = 7,81	✓✓answer (2)
1.3	The one standard deviation limits are $(\bar{x} - 1\sigma ; \bar{x} + 1\sigma)$ = (21,47 – 7,81; 21,47 + 7,81) = (13,66 ; 29,28) ∴ 13 people lie within 1 standard deviation of the mean.	✓ interval ✓13 people (2)
1.4	5 12 13 15 **18** 18 18 19 20 **21** 21 22 23 23 **26** 29 33 35 37 IQR = 26 – 18 = 8	✓ $Q_1 = 18$ ✓ $Q_3 = 26$ ✓IQR = 8 (3)
1.5		✓✓box ✓whiskers (3)
1.6	There is a marked difference between the lowest value (5) and the next lowest value (12) whilst the differences between all other data points are within at most 3 values. ∴ 5 is an outlier	✓reason ✓5 is an outlier (2) **[14]**

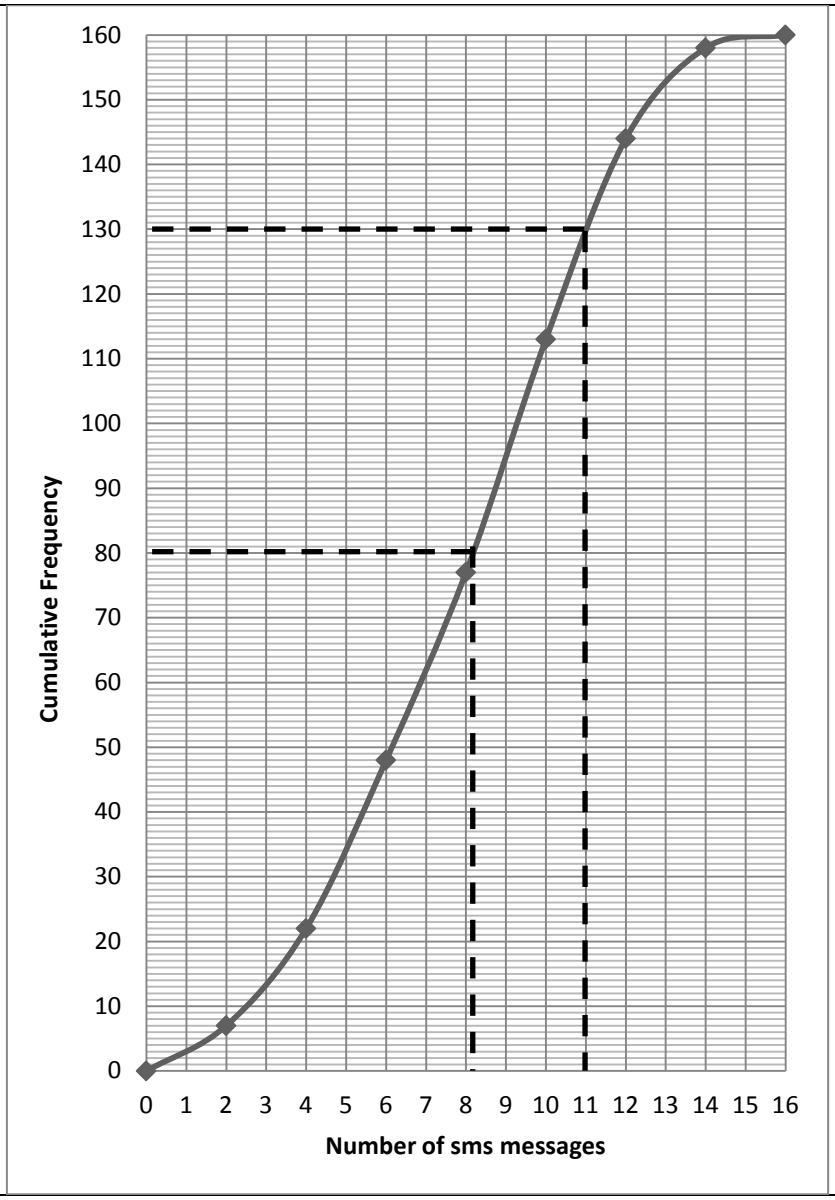
QUESTION 2

2.1

Class	Frequency	Cumulative frequency
$0 \le m < 2$	7	7
$2 \le m < 4$	15	22
$4 \le m < 6$	26	48
$6 \le m < 8$	29	77
$8 \le m < 10$	36	113
$10 \le m < 12$	31	144
$12 \le m < 14$	14	158
$14 \le m < 16$	2	160

✓ first three cumulative frequencies correct
✓remainder correct (total = 160)
(2)

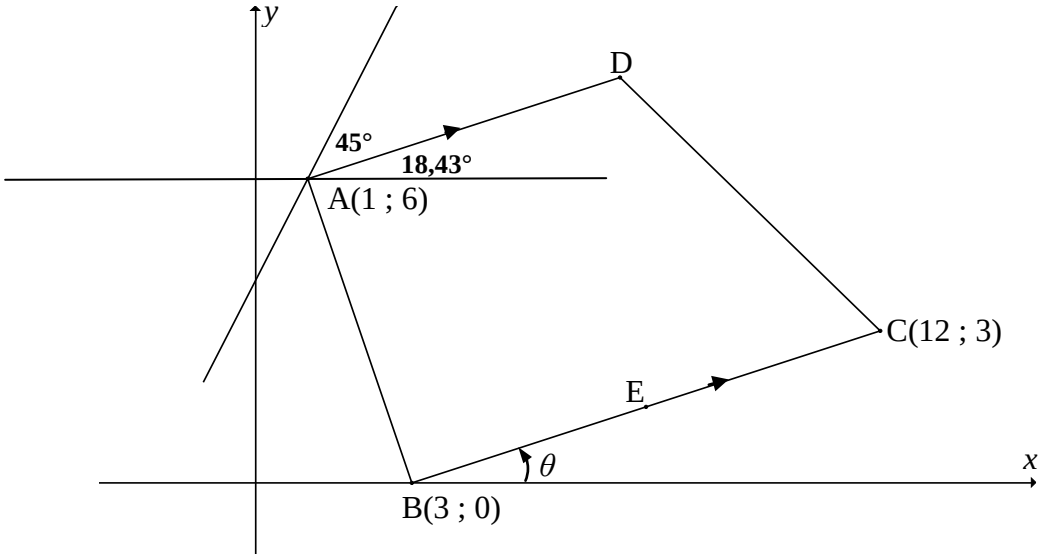
2.2



✓grounding at 0
✓plotting cumulative frequencies at upper limits
✓ smooth shape of curve
(3)

2.3	The median for the data is approximately 8 messages.	✓Median (1)
2.4	Approximately 130 learners sent 11 or fewer messages. Therefore 30 learners sent more than 11 messages. $\frac{30}{160} \times 100\% = 18{,}75\%$	✓30 learners ✓answer (2)
2.5	Skewed to the left or negatively skewed	✓answer (1) **[9]**

QUESTION 3



3.1	$E\left(\frac{3+12}{2};\frac{0+3}{2}\right)$ $=\left(7\frac{1}{2};1\frac{1}{2}\right)$	✓ substitution into midpoint formula ✓answer (2)
3.2	$m_{BC}=\frac{3-0}{12-3}$ $=\frac{1}{3}$	✓ substitution into gradient formula ✓answer (2)
3.3	$\tan\theta = m_{BC}=\frac{1}{3}$ $\theta=\tan^{-1}\left(\frac{1}{3}\right)=18{,}43°$	✓ $\tan\theta = m_{BC}$ ✓answer (2)
3.4	$m_{AD}=m_{BC}=\frac{1}{3}$ AD||BC, equal gradients $m_{AB}=\frac{6-0}{1-3}=-3$ $\therefore m_{AD}\times m_{AB}=\frac{1}{3}\times -3=-1$ $\therefore$ AD ⊥ AB	✓ $m_{AD}=\frac{1}{3}$ ✓ $m_{AB}=-3$ ✓ $m_{AD}\times m_{AB}=-1$ (3)
3.5	inclination of new line = 45° + 18,43° = 63,43° $\therefore$ tan 63,43° = 2 = m_{line} $\therefore\ y-6=2(x-1)$ $y=2x+4$	✓18,43° ✓63,43° ✓$m = 2$ ✓subst of (1 ; 6) ✓ equation (5) **[14]**

QUESTION 4

4.1	$m_{QP} = m_{OS} = 6$ QP‖OS, equal gradients $y - 17 = 6(x + 3)$ $y = 6x + 35$	✓ $m_{QP} = 6$ ✓subst (–3 ; 17) into formula ✓equation (3)
4.2	$6x + 35 = -x$ $7x = -35$ $x = -5$ $y = -(-5) = 5$ ∴ Q(–5 ; 5) **OR** $y = 6(-5) + 35 = 5$	✓ setting up equation ✓$x = -5$ ✓ $y = 5$ ✓coordinates of Q (4)
4.3	$OQ^2 = (-5 - 0)^2 + (5 - 0)^2$ $= 50$ $OQ = \sqrt{50} = 5\sqrt{2}$ *units*	✓ substitution into distance formula ✓$5\sqrt{2}$ (2)
4.4	$m_{OS} = 6$ ∴ inclination of OS is $\tan^{-1}(4) = 80{,}54°$ $m_{OQ} = -1$ ∴ inclination of QO is $180° - \tan^{-1}(1) = 135°$ $\alpha = 135° - 80{,}54...°$ $= 54{,}46°$	✓80,54° ✓135° ✓54,46° (3)
4.5	$QS^2 = OS^2 + OQ^2 - 2OS.OQ.\cos\alpha$ $= 148 + 50 - 2(\sqrt{148})(\sqrt{50}.\cos 54{,}46°$ $QS = 9{,}90$ units	✓correct use of cosine rule ✓substitution into formula ✓9,90 (3) **[15]**

QUESTION 5

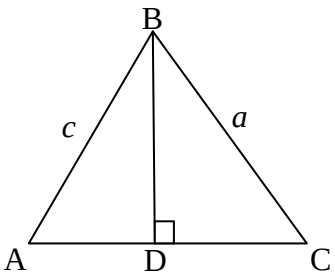
5.1.1	$\cos\alpha = -\frac{5}{13}$	✓ $-\frac{5}{13}$ (1)
5.1.2	$(-5)^2 + b^2 = 13^2$ $b^2 = 169 - 25 = 144$ $b = 12$ $\tan(180° - \alpha)$ $= -\tan\alpha$ $= -(-\frac{12}{5})$ $= \frac{12}{5}$	✓$b = 12$ ✓$-\tan\alpha$ ✓$\frac{12}{5}$ (3)
5.2.1	$\frac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$ $= \frac{\sin\theta\ \cos\theta\ (-\tan\theta)}{-\sin\theta}$ $= -\cos\theta\left(-\frac{\sin\theta}{\cos\theta}\right)$ $= \sin\theta$	✓✓✓ reductions ✓$\tan\theta = \frac{\sin\theta}{\cos\theta}$ ✓$\sin\theta$ (5)
5.2.2	From 5.2.1: $\sin\theta = 0{,}5$ Ref $\angle = 30°$ $\therefore \theta = 30°$ or $\theta = 150°$	✓$\sin\theta = 0{,}5$ ✓$30°$ ✓$150°$ (3)

5.3.1	$LHS = \frac{8}{\sin^2 A} - \frac{4}{1+\cos A}$ $= \frac{8}{1-\cos^2 A} - \frac{4}{1+\cos A}$ $= \frac{8}{(1-\cos A)(1+\cos A)} - \frac{4}{1+\cos A}$ $= \frac{8-4(1-\cos A)}{(1-\cos A)(1+\cos A)}$ $= \frac{8-4+4\cos A}{(1-\cos A)(1+\cos A)}$ $= \frac{4(1+\cos A)}{(1-\cos A)(1+\cos A)}$ $= \frac{4}{1-\cos A} = RHS$	✓ $\sin^2 A = 1-\cos^2 A$ ✓factorising ✓ addition ✓simplification ✓ factorising (5)
5.3.2	Identity is undefined when $\sin^2 A = 0$. That is when $\sin A = 0$ or cosA = ±1 ∴ A = 0° or A = 180° or A = 360°.	✓✓✓ each value (3)
5.4	$8\cos^2 x - 2\cos x - 1 = 0$ $(4\cos+1)(2\cos x - 1) = 0$ $\cos x = -\frac{1}{4}$ or $\cos x = \frac{1}{2}$ $\therefore x = 104{,}48° + k.360°; k \in Z$ or $x = 60° + k.360°; k \in Z$ $x = 255{,}52° + k.360°; k \in Z$ $x = 300° + k.360°; k \in Z$	✓ factorising ✓values of cos*x* ✓ 104,48° or 255,52° ✓ 60° or 300° ✓ + 360°.*k* ✓*k*ε Z (6) **[26]**

QUESTION 6

6.1	$p = -45°$ $q = -1$	✓ value of p ✓ value of q (2)
6.2	B(157,5° ; – 0,38)	✓ value of x ✓ value of y (2)
6.3	$f(x) < g(x)$ when $-180° \le x < -22,5°$ or $157,5° < x \le 180°$	✓ $-180° \le x < -22,5°$ ✓ $157,5° < x \le 180°$ (2)
6.4.1	$h(x) = \cos(x - 45° + 30°)$ $= \cos(x - 15°)$	✓ + 30° ✓ simplest form (2)
6.4.2	$x = -135° - 30° = -165°$	✓ –165° (1) **[9]**

QUESTION 7

7.1	Draw BD ⊥ AC In ΔABD: $\sin A = \frac{BD}{c} \therefore BD = c.\sin A$ In ΔCBD: $\sin C = \frac{BD}{a} \therefore BD = a.\sin C$ $\therefore c.\sin A = a.\sin C$ $\therefore \frac{\sin A}{a} = \frac{\sin C}{c}$ 	✓ construction ✓ sin A ✓ making BD the subject ✓ sin C ✓$c.\sin A = a.\sin C$ (5)
7.2.1	$\frac{\sin R}{r} = \frac{\sin P}{p}$ $\frac{\sin R}{27,2} = \frac{\sin 132°}{73,2}$ $\sin R = \frac{27,2 \times \sin 132°}{73,2}$ $= 0,276...$ $\hat{R} = 16,03°$	✓ substitution into correct formula ✓making sin R the subject ✓16,03° (3)

7.2.2	$\hat{Q} = 180° - 132° - 16{,}03° = 31{,}97°$ area of PQR $= \frac{1}{2} pr.\sin Q$ $= \frac{1}{2}(73{,}2)(27{,}2).\sin 31{,}97°$ $= 527{,}10\ \text{cm}^2$	✓ $\hat{Q} = 31{,}97°$ ✓ substitution into correct formula ✓ 527,1 (3)
7.3.1	$P\hat{S}Q = 180° - (a+b)$ In ΔPSQ: $\frac{SQ}{\sin P} = \frac{PQ}{\sin P\hat{S}Q}$ $\frac{SQ}{\sin a} = \frac{h}{\sin[180° - (a+b)]}$ $\frac{SQ}{\sin a} = \frac{h}{\sin(a+b)}$ $SQ = \frac{h\sin a}{\sin(a+b)}$	✓ $P\hat{S}Q = 180° - (a+b)$ ✓ $\sin[180° - (a+b)]$ = sin $(a + b)$ ✓making SQ the subject (3)
7.3.2	$S\hat{Q}R = 90° - b$ In ΔRSQ: $\frac{RS}{SQ} = \sin S\hat{Q}R$ $RS = SQ.\sin(90° - b)$ $= \frac{h\sin a}{\sin(a+b)}.\cos b$ $= \frac{h\sin a.\cos b}{\sin(a+b)}$	✓ $S\hat{Q}R = 90° - b$ ✓ use sine ratio correctly ✓ $\sin(90° - b)$ = cos$b$ (3) **[17]**

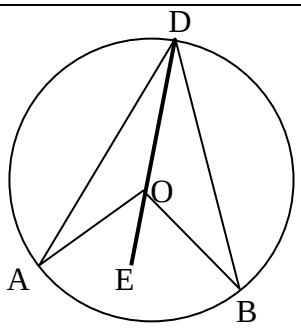
QUESTION 8

	Volume of hemisphere $= \frac{1}{2}\left[\frac{4}{3}\pi r^3\right]$ $= \frac{2}{3}\pi (3)^3$ $= 18\pi \text{ cm}^3$ Volume of conical hole $= \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(1{,}5)^2(\frac{8}{9})$ $= \frac{2}{3}\pi \quad cm^3$ $\therefore \frac{\text{volume of metal A}}{\text{volume of metal B}} = \frac{17\frac{1}{3}\pi}{\frac{2}{3}\pi} = \frac{26}{1}$ Ratio of volume metal A : Volume metal B = 26 : 1	✓ substitution into correct formula ✓18 π ✓ substitution into correct formula ✓$\frac{2}{3}\pi$ ✓$17\frac{1}{3}\pi$ ✓ratio 26 : 1 (6) **[6]**

QUESTION 9

9.1	…bisects the chord.	✓ answer (1)
9.2.1	OE = 10 cm … O midpoint of DE OC = OE – CE = 10 – 2 = 8 cm	✓ OE = 10 ✓ OC = 8 (2)
9.2.2	In ΔCOQ: $QC^2 = OQ^2 – OC^2$ … Theorem of Pythagoras $= (10)^2 – (8)^2$ = 36 QC = 6 cm ∴PQ = 2QC … line drawn from centre ⊥ to chord bisects chord PQ = 12 cm	✓Using Theorem of Pythagoras ✓ QC = 6 ✓ PQ = 12 (S) ✓ reason (4) **[7]**

QUESTION 10

10.1	 Construction: Produce DO to E Proof: In ΔOBD: $O\hat{B}D = O\hat{D}B$ …OD = OB = r $E\hat{O}B = 2 \times O\hat{D}B$ …exterior angle of triangle In ΔAOD: $O\hat{A}D = O\hat{D}A$ …OA = OD = r $E\hat{O}A = 2 \times O\hat{D}A$ …exterior angle of triangle $A\hat{O}B = E\hat{O}B + E\hat{O}A$ $= 2 \times O\hat{D}B + 2 \times O\hat{D}A$ $= 2(O\hat{D}B + O\hat{D}A)$ $= 2A\hat{D}B$	✓ construction ✓ $O\hat{B}D = O\hat{D}B$ ✓ $E\hat{O}B = 2 \times O\hat{D}B$ (S/R) ✓ $E\hat{O}A = 2 \times O\hat{D}A$ (S/R) ✓ $A\hat{O}B = E\hat{O}B + E\hat{O}A$ (5)
10.2.1(a)	$\hat{M} = 76°$ … ∠ at centre = 2(∠ at circumference)	✓ 76° ✓ reason (2)
10.2.1(b)	$\hat{T}_2 = 38°$ …ext∠ of cyc quad KTAB	✓ 38° ✓ reason (2)
10.2.1(c)	$\hat{C} = 38°$ … ext∠ of cyclic quad or $\angle^s$ in same segment	✓ 38° ✓ reason (2)
10.2.1(d)	$C\hat{A}N = \hat{C} = 38°$ …NA = NC $\hat{K}_4 = 38°$ …ext ∠ of cyclic quad CATK	✓ $C\hat{A}N = 38°$ (S/R) ✓ $\hat{K}_4 = 38°$ (2)
10.2.2	$\therefore \hat{K}_4 = \hat{T}_2$ ∴NK = NT …base $\angle^s$ equal	✓ statement ✓ reason (2)
10.2.3	$\hat{N} = 180° - (38° + 38°)$ …$\angle^s$ of ΔKNT $= 104°$ $\hat{N} + K\hat{M}A = 104° + 76° = 180°$ ∴ AMKN is cyclic quad …opposite $\angle^s$ = 180°	✓ $\hat{N} = 104°$ (S/R) ✓ $\hat{N} + K\hat{M}A = 180°$ ✓reason (3) **[18]**

QUESTION 11

11.1	..... equal to the angle subtended by the same chord in the alternate segment.	✓ alternate segment (1)
11.2.1	$\hat{A}_1 = \hat{C}_2 = x$ …tangent chord theorem $\hat{C}_2 = \hat{G}_2 = x$ …tangent chord theorem $\therefore \hat{A}_1 = \hat{G}_2 = x$ $\therefore$ BCG $\|\|$ EA …alternate $\angle^s$ =	✓ $\hat{A}_1 = \hat{C}_2 = x$ ✓ reason ✓ $\hat{C}_2 = \hat{G}_2 = x$ ✓ reason ✓ conclusion with reason (5)
11.2.2	$\hat{E}_1 = \hat{C}_3 = y$ …alternate $\angle^s$; BG $\|\|$ EA $\hat{F}_1 = \hat{C}_3 = y$ …ext$\angle$ of cyclic quad CDFG $\therefore \hat{E}_1 = \hat{F}_1 = y$ $\therefore$ EA is a tangent …converse tangent-chord theorem	✓ $\hat{E}_1 = \hat{C}_3 = y$ (S/R) ✓ $\hat{F}_1 = \hat{C}_3 = y$ (S) ✓ reason ✓ $\hat{E}_1 = \hat{F}_1 = y$ ✓reason (5)
11.2.3	$\hat{B} = C\hat{A}E$ …tangent-chord theorem $\hat{C}_1 = C\hat{A}E$ … alternate $\angle^s$; BG $\|\|$ EA $\hat{C}_1 = \hat{B}$ $\therefore$ AB = AC …base $\angle^s$ =	✓ $C\hat{A}E = \hat{B}$ ✓reason ✓ $\hat{C}_1 = C\hat{A}E$ (S/R) ✓ reason (4) **[15]**

TOTAL: 150

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